Write your name here

| Surname | Other names |
| --- | --- |
| | |

**Pearson Edexcel Level 3 GCE**

| Centre Number | Candidate Number |
| --- | --- |
| | |

# Further Mathematics

## Advanced
## Further Mathematics Option 2
## Paper 4: Further Mechanics 2

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 30 minutes**

Paper Reference
**9FM0/4F**

**You must have:**
Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

S54446A

1/1/1/
S 5 4 4 4 6 A 0 1 2 3

Pearson

**Answer ALL questions. Write your answers in the spaces provided.**

Unless otherwise indicated, whenever a numerical value of $g$ is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

**1.** A flag pole is 15 m long.

The flag pole is non-uniform so that, at a distance $x$ metres from its base, the mass per unit length of the flag pole, $m\,\mathrm{kg\,m^{-1}}$ is given by the formula $m = 10\left(1 - \frac{x}{25}\right)$.

The flag pole is modelled as a rod.

(a) Show that the mass of the flag pole is 105 kg.

**(3)**

(b) Find the distance of the centre of mass of the flag pole from its base.

**(4)**

**Question 1 continued**

**(Total for Question 1 is 7 marks)**

**2.**

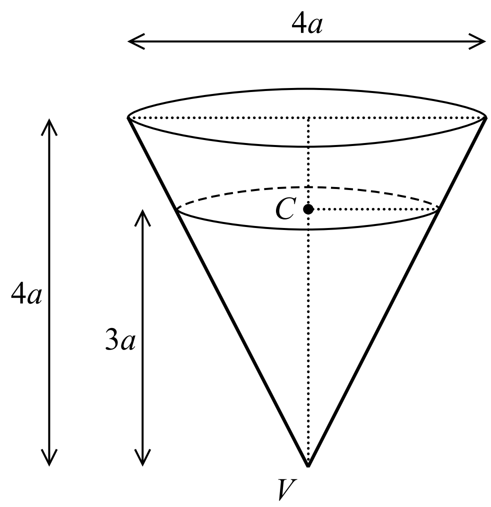



**Figure 1**

A hollow right circular cone, of base diameter $4a$ and height $4a$ is fixed with its axis vertical and vertex $V$ downwards, as shown in Figure 1.

A particle of mass $m$ moves in a horizontal circle with centre $C$ on the rough inner surface of the cone with constant angular speed $\omega$.

The height of $C$ above $V$ is $3a$.

The coefficient of friction between the particle and the inner surface of the cone is $\frac{1}{4}$.

Find, in terms of $a$ and $g$, the greatest possible value of $\omega$.

**(8)**

**Question 2 continued**

**(Total for Question 2 is 8 marks)**

**3.**

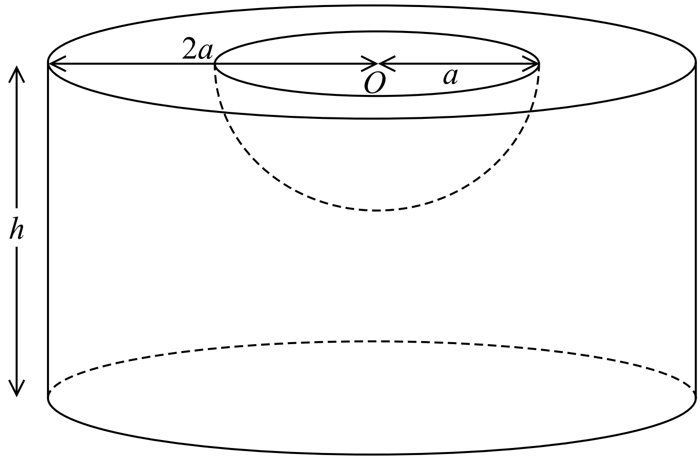



**Figure 2**

A uniform solid cylinder has radius $2a$ and height $h$ ($h > a$).

A solid hemisphere of radius $a$ is removed from the cylinder to form the vessel $V$.

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The centre $O$ of the hemisphere is also the centre of the upper plane face of the cylinder, as shown in Figure 2.

(a) Show that the centre of mass of $V$ is $\dfrac{3(8h^2 - a^2)}{8(6h - a)}$ from $O$.

**(5)**

The vessel $V$ is placed on a rough plane which is inclined at an angle $\phi$ to the horizontal.

The lower plane circular face of $V$ is in contact with the inclined plane.

Given that $h = 5a$, the plane is sufficiently rough to prevent $V$ from slipping and $V$ is on the point of toppling,

(b) find, to three significant figures, the size of the angle $\phi$.

**(4)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 9 marks)**

**4.** A car of mass 500 kg moves along a straight horizontal road.

The engine of the car produces a constant driving force of 1800 N.

The car accelerates from rest from the fixed point $O$ at time $t = 0$ and at time $t$ seconds the car is $x$ metres from $O$, moving with speed $v\,\text{m s}^{-1}$.

When the speed of the car is $v\,\text{m s}^{-1}$, the resistance to the motion of the car has magnitude $2v^2$ N.

At time $T$ seconds, the car is at the point $A$, moving with speed $10\,\text{m s}^{-1}$.

(a) Show that $T = \dfrac{25}{6}\ln 2$

**(6)**

(b) Show that the distance from $O$ to $A$ is $125\ln\dfrac{9}{8}$ m.

**(5)**

**Question 4 continued**

**(Total for Question 4 is 11 marks)**

**5.**

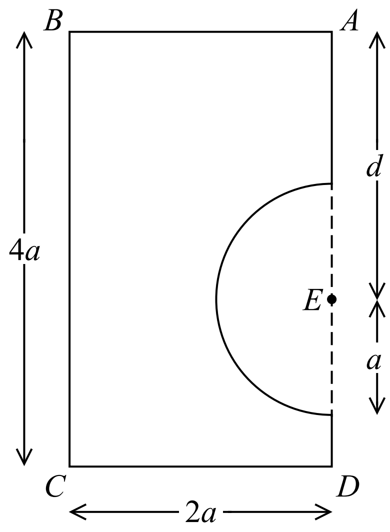



**Figure 3**

A shop sign is modelled as a uniform rectangular lamina *ABCD* with a semicircular lamina removed.

The semicircle has radius $a$, $BC = 4a$ and $CD = 2a$.

The centre of the semicircle is at the point $E$ on $AD$ such that $AE = d$, as shown in Figure 3.

(a) Show that the centre of mass of the sign is $\dfrac{44a}{3(16-\pi)}$ from $AD$.

**(4)**

The sign is suspended using vertical ropes attached to the sign at $A$ and at $B$ and hangs in equilibrium with $AB$ horizontal.

The weight of the sign is $W$ and the ropes are modelled as light inextensible strings.

(b) Find, in terms of $W$ and $\pi$, the tension in the rope attached at $B$.

**(2)**

The rope attached at $B$ breaks and the sign hangs freely in equilibrium suspended from $A$, with $AD$ at an angle $\alpha$ to the downward vertical.

Given that $\tan \alpha = \dfrac{11}{18}$

(c) find $d$ in terms of $a$ and $\pi$.

**(6)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 12 marks)**

6. A small bead $B$ of mass $m$ is threaded on a circular hoop.

The hoop has centre $O$ and radius $a$ and is fixed in a vertical plane.

The bead is projected with speed $\sqrt{\frac{7}{2}ga}$ from the lowest point of the hoop.

The hoop is modelled as being smooth.

When the angle between $OB$ and the downward vertical is $\theta$, the speed of $B$ is $v$.

(a) Show that $v^2 = ga\left(\frac{3}{2} + 2\cos\theta\right)$

**(3)**

(b) Find the size of $\theta$ at the instant when the contact force between $B$ and the hoop is first zero.

**(5)**

(c) Give a reason why your answer to part (b) is not likely to be the actual value of $\theta$.

**(1)**

(d) Find the magnitude and direction of the acceleration of $B$ at the instant when $B$ is first at instantaneous rest.

**(5)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 14 marks)**

**7.** Two points $A$ and $B$ are 6 m apart on a smooth horizontal surface.

A light elastic string of natural length 2 m and modulus of elasticity 20 N, has one end attached to the point $A$.

A second light elastic string of natural length 2 m and modulus of elasticity 50 N, has one end attached to the point $B$.

A particle $P$ of mass 3.5 kg is attached to the free end of each string.

The particle $P$ is held at the point on $AB$ which is 2 m from $B$ and then released from rest.

In the subsequent motion both strings remain taut.

(a) Show that $P$ moves with simple harmonic motion about its equilibrium position.

**(7)**

(b) Find the maximum speed of $P$.

**(2)**

(c) Find the length of time within each oscillation for which $P$ is closer to $A$ than to $B$.

**(5)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 14 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

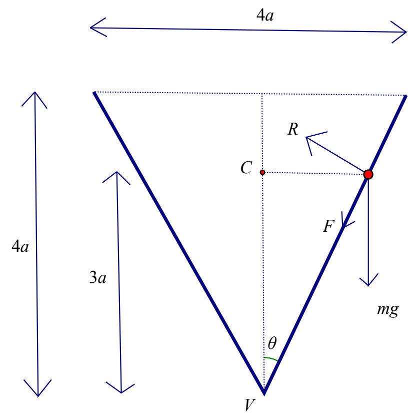
## Paper 4F: Further Mechanics 2 Mark Scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 1(a) | Total mass $= \int_0^{15} 10\left(1-\frac{x}{25}\right) dx$ | M1 | 2.1 |
| | $= \left[10x - \frac{x^2}{5}\right]_0^{15}$ | A1 | 1.1b |
| | $= 150 - \frac{225}{5} = 105$ (kg) * | A1* | 1.1b |
| | | (3) | |
| (b) | Taking moments about the base: $\int_0^{15} 10x\left(1-\frac{x}{25}\right) dx$ | M1 | 3.4 |
| | $= \left[5x^2 - \frac{2}{15}x^3\right]_0^{15}$ $(= 675)$ | A1 | 1.1b |
| | $\Rightarrow 105d = 675$ | M1 | 3.4 |
| | $d = 6.43$ (m) $6\frac{3}{7}$ (m) | A1 | 1.1b |
| | | (4) | |
| | | | (7 marks) |

**Notes:**

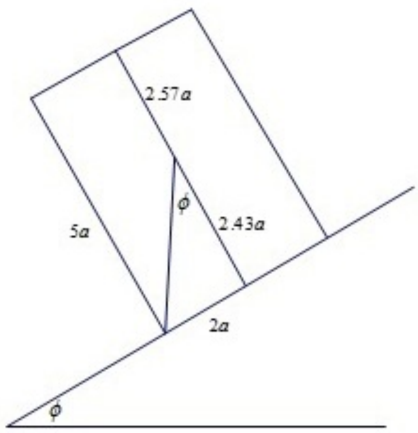
**(a)**
**M1:** Use integration (usual rules)
**A1:** Correct integration
**A1*:** Use limits and show sufficient working to justify given answer

**(b)**
**M1:** Use the model to find the moment about the base (usual rules for integration)
**A1:** Correct integration
**M1:** Use the model to complete the moments equation
Require 105 and their 675 used correctly
**A1:** 6.43 or better

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 2 |   | | |
| | Complete overall strategy | M1 | 3.1b |
| | Resolve vertically | M1 | 3.3 |
| | $mg + F\cos\theta = R\sin\theta$ | A1 | 1.1b |
| | Horizontal equation of motion | M1 | 3.3 |
| | $mr\omega^2 = R\cos\theta + F\sin\theta$ | A1 | 1.1b |
| | Use of limiting friction since maximum $\omega$ | M1 | 3.3 |
| | Substitute for trig ratios: $\frac{3a\omega^2}{2g} = \frac{9}{2}$ | M1 | 1.1b |
| | Maximum $\omega = \sqrt{\frac{3g}{a}}$ | A1 | 1.1b |
| | | | **(8 marks)** |

**Notes:**

**M1:** Overall strategy to form equation in $\omega$ only e.g. consider vertical and horizontal motrion and limiting friction
**M1:** Needs all 3 terms. Condone sign errors and sin/cos confusion
**A1:** Correct unsimplified equation
**M1:** Needs all 3 terms. Condone sign errors and sin/cos confusion
**A1:** Correct unsimplified equation
**M1:** Seen or implied
**M1:** Substitute to achieve equation in $a$, $\omega$ and $g$ only
**A1:** Or equivalent exact form

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 3(a) | <table><tr><td></td><td>mass</td><td>c of m from $O$</td></tr><tr><td>cylinder</td><td>$4\pi a^2 h$</td><td>$\frac{h}{2}$</td></tr><tr><td>hemisphere</td><td>$\frac{2}{3}\pi a^3$</td><td>$\frac{3}{8}a$</td></tr><tr><td>$V$</td><td>$4\pi a^2 h - \frac{2}{3}\pi a^3$</td><td>$d$</td></tr></table> | | |
| | Mass ratios | B1 | 1.2 |
| | Correct distances | B1 | 1.2 |
| | Moments about a diameter through $O$ | M1 | 2.1 |
| | $4\pi a^2 h \times \frac{h}{2} - \frac{2}{3}\pi a^3 \times \frac{3}{8}a = 2\pi a^2\left(2h - \frac{1}{3}a\right) \times d$ | A1 | 1.1b |
| | $d = \dfrac{h^2 - \frac{a^2}{8}}{2h - \frac{a}{3}} = \dfrac{3(8h^2 - a^2)}{8(6h - a)}$ * | A1* | 2.2a |
| | | **(5)** | |
| **(b)** |  | | |
| | $h = 5a \Rightarrow d = 2.573...a$ | B1 | 1.1b |
| | About to topple so c of m above tipping point | M1 | 2.2a |
| | $\Rightarrow \tan\phi = \dfrac{2a}{5a - 2.573a}$ | A1ft | 1.1b |
| | $\phi = 39.5°$ **or** 0.689 rads | A1 | 1.1b |
| | | **(4)** | |
| | | | **(9 marks)** |

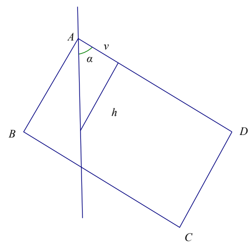
| **Question 3 notes:** |
|---|
| **(a)**<br>**B1:** Correct mass ratios<br>**B1:** Correct distances<br>**M1:** All three terms & dimensionally correct. Could use a parallel axis but final answer must be for the distance from $O$<br>**A1:** Correct unsimplified equation<br>**A1*:** Deduce the given answer. Their working must make it clear how they reached their answer |
| **(b)**<br>**B1:** Distance of com from base<br>**M1:** Condone tan the wrong way up<br>**A1ft:** Correct unsimplified expression for trig ratio for $\phi$ following their $d$<br>**A1:** $39.5°$ or 0.689 rads |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 4(a) | Equation of motion: $1800 - 2v^2 = 500a$ (when seen) | B1 | 2.1 |
| | Select form for $a$: $= 500\frac{dv}{dt}$ | M1 | 2.5 |
| | $\int \frac{2}{500}dt = \int \frac{1}{900 - v^2}dv = \frac{1}{60}\int \frac{1}{30+v} + \frac{1}{30-v}dv$ | M1 | 2.1 |
| | $\frac{t}{250} = \frac{1}{60}\ln(30+v) - \frac{1}{60}\ln(30-v) \ (+C)$ | A1 | 1.1b |
| | $T = \frac{25}{6}\ln\left(\frac{30+10}{30-10}\right) = \frac{25}{6}\ln 2$ * | M1<br>A1* | 2.1<br>2.2a |
| | | **(6)** | |
| (b) | Equation of motion: $500v\frac{dv}{dx} = 1800 - 2v^2$ | M1 | 2.5 |
| | $\int \frac{500v}{1800 - 2v^2}dv = \int 1dx$ | M1 | 2.1 |
| | $-125\ln(1800 - 2v^2) = x \ (+C)$ | A1 | 1.1b |
| | Use boundary conditions: $x = -125 \ln 1600 + 125 \ln 1800$ | M1 | 2.1 |
| | $x = 125\ln\frac{9}{8}$ (m) * | A1* | 2.2a |
| | | **(5)** | |
| | | **(11 marks)** | |

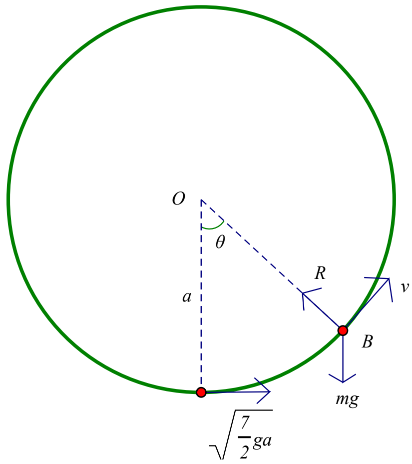
**Notes:**

**(a)**
**B1:** All three terms & dimensionally correct
**M1:** Use of correct form for acceleration to give equation in $v$, $t$ only
**M1:** Separate variables and integrate
**A1:** Condone missing $C$
**M1:** Use boundary conditions correctly
**A1*:** Show sufficient working to justify given answer and a 'statement' that the required result has been achieved

**(b)**
**M1:** Correct form of acceleration in the equation of motion to give equation in $v$, $x$ only
**M1:** Separate variables and integrate
**A1:** Condone missing $C$
**M1:** Extract and use boundary conditions
**A1*:** Show sufficient working to justify given answer and a 'statement' that the required result has been achieved

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 5(a) | <table><tr><td></td><td>Mass</td><td>From *AD*</td></tr><tr><td>Rectangle</td><td>$8a^2$</td><td>$a$</td></tr><tr><td>Semicircle</td><td>$\frac{1}{2}\pi a^2$</td><td>$\frac{4a}{3\pi}$</td></tr><tr><td>Sign</td><td>$a^2\left(8-\frac{\pi}{2}\right)$</td><td>$h$</td></tr></table> | | |
| | Mass ratios | B1 | 1.2 |
| | Moments about *AD* | M1 | 2.1 |
| | $a^2\left(8-\frac{\pi}{2}\right)h=8a^2\times a-\frac{1}{2}\pi a^2\times\frac{4a}{3\pi}\left(=8a^3-\frac{2}{3}a^3=\frac{22}{3}a^3\right)$ | A1 | 1.1b |
| | $\Rightarrow h=\frac{22}{3}a\div\left(8-\frac{\pi}{2}\right)=\frac{44a}{3(16-\pi)}$ * | A1* | 2.2a |
| | | **(4)** | |
| (b) | Moments about *A*<br>$2aT=\frac{44a}{3(16-\pi)}W$ | M1 | 3.1b |
| | $T=\frac{hW}{2a}=\frac{22W}{3(16-\pi)}$ | A1 | 1.1b |
| | | **(2)** | |
| (c) |  | | |
| | Take moments about *AB* to find distance of com from *AB* | M1 | 3.1b |
| | $8a^2\times 2a-\frac{1}{2}\pi a^2\times d=\left(8-\frac{1}{2}\pi\right)a^2\times v$ | A1 | 1.1b |
| | $v=\frac{32a-\pi d}{16-\pi}$ | A1 | 1.1b |
| | Correct trig for the given angle | M1 | 3.1b |
| | $\tan\alpha=\frac{11}{18}=\frac{h}{v}=\frac{44a}{3(32a-\pi d)}$ | A1ft | 1.1b |
| | $(24a=32a-\pi d,\quad 8a=\pi d)\quad d=\frac{8a}{\pi}$ | A1 | 1.1b |
| | | **(6)** | |
| | | | **(12 marks)** |

| **Question 5 notes:** | |
|---|---|
| **(a)** | |
| **B1:** | Correct mass ratios |
| **M1:** | Need all three terms, must be dimensionally correct |
| **A1:** | Correct unsimplified equation |
| **A1*:** | Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved |
| **(b)** | |
| **M1:** | Could also take moments about B **or** about the c.o.m. and use |
| **A1:** | cso |
| **(c)** | |
| **M1:** | All terms and dimensionally correct |
| **A1:** | Correct unsimplified equation |
| **A1:** | Or equivalent |
| **M1:** | Condone tan the wrong way up |
| **A1:** | Equation in a and d; follow through on their v |
| **A1:** | cao |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6(a) |  | | |
| | Conservation of energy | M1 | 2.1 |
| | $\frac{1}{2}mv^2 + mga(1-\cos\theta) = \frac{1}{2}m\left(\frac{7}{2}ga\right)$ | A1 | 1.1b |
| | $v^2 = ga\left(\frac{3}{2} + 2\cos\theta\right)$ * | A1* | 2.2a |
| | | **(3)** | |
| (b) | Resolve parallel to $OB$ and use $\frac{mv^2}{a}$ | M1 | 3.1b |
| | $R - mg\cos\theta = \frac{mv^2}{a}$ | A1 | 1.1b |
| | Use R= 0 $\quad g\cos\theta = -\frac{v^2}{a}$ | M1 | 3.1b |
| | Solve for $\theta$ $\Rightarrow$ $g\cos\theta = -g\left(\frac{3}{2} + 2\cos\theta\right)$ | M1 | 1.1b |
| | $\theta = 120°$ | A1 | 1.1b |
| | | **(5)** | |
| (c) | Any appropriate comment e.g. the hoop is unlikely to be smooth | B1 | 3.5b |
| | | **(1)** | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6(d) | At rest $\Rightarrow \quad v=0$ | M1 | 3.1b |
| | $\Rightarrow \quad \cos\theta = -\frac{3}{4}$ | A1 | 1.1b |
| | Acceleration is tangential | M1 | 3.1b |
| | Magnitude $\left\|g\cos(\theta-90)\right\| = 6.48$ m s$^{-2}$ or $\frac{\sqrt{7}}{4}g$ | A1 | 1.1b |
| | At $\left(\cos^{-1}\left(-\frac{3}{4}\right)-90=\right)48.6°$ to the downward vertical | A1 | 1.1b |
| | | **(5)** | |
| | | **(14 marks)** | |

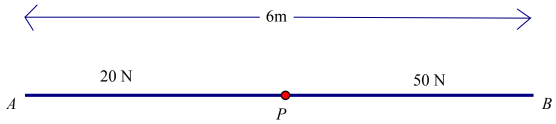
**Question 6 notes:**

**(a)**
**M1:** All terms required. Must be dimensionally correct
**A1:** Correct unsimplified equation
**A1*:** Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved

**(b)**
**M1:** Resolve parallel to $OB$
**A1:** Correct equation
**M1:** Use $R=0$ seen or implied
**M1:** Solve for $\theta$
**A1:** Accept $\frac{2\pi}{3}$

**(c)**
**B1:** Any appropriate comment e.g.
- hoop may not be smooth;
- air resistance could affect the motion

**(d)**
**M1:** $v=0$ seen or implied
**A1:** Correct equation in $\theta$
**M1:** Correct direction for acceleration
**A1:** Accept 6.48, 6.5 or exact in $g$
**A1:** Accept 0.848 (radians)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(a) |   | | |
| | $T_A = \frac{20e}{2}$, $T_B = \frac{50(2-e)}{2}$ $e$ | M1 | 3.1a |
| | In equilibrium $T_A = T_B$ , $10e = 25(2-e)$ | M1 | 3.1a |
| | $(35e = 50)$, $e = \frac{10}{7}$ | A1 | 1.1b |
| | Equation of motion for $P$ when distance $x$ from equilibrium position towards $B$: | M1 | 3.1a |
| | $3.5\ddot{x} = T_B - T_A = \frac{50(2-e-x)}{2} - \frac{20(e+x)}{2}$ | A1<br>A1 | 1.1b<br>1.1b |
| | $= \frac{50\left(\frac{4}{7}-x\right)}{2} - \frac{20\left(\frac{10}{7}+x\right)}{2}$ | | |
| | $\Rightarrow 3.5\ddot{x} = -35x$, $\ddot{x} = -10x$<br>and hence SHM about the equilibrium position | A1 | 3.2a |
| | | **(7)** | |
| (b) | Amplitude $= 2 - \frac{10}{7} = \frac{4}{7}$ | B1 ft | 2.2a |
| | Use of max speed $= a\omega$ | M1 | 1.1b |
| | $= \frac{4}{7}\sqrt{10} = 1.81$ (m s$^{-1}$) | A1 ft | 1.1b |
| | | **(3)** | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(c) | Nearer to $A$ than to $B$: $x < -\frac{3}{7}$ | B1 | 3.1a |
| | Solve for $\sqrt{10}t$ : $\cos\sqrt{10}t = -\frac{3}{4}$ , $\sqrt{10}t = 2.418$............ | M1 | 3.1a |
| | Length of time: $\frac{2}{\sqrt{10}}(\pi - 2.418...)$ | M1 | 1.1b |
| | 0.457 (seconds) | A1 | 1.1b |
| | Alternative: $\frac{3.864 - 2.419}{\sqrt{10}} = 0.457$ | | |
| | Alternative:<br>$x = \frac{4}{7}\sin\sqrt{10}t = \frac{3}{7} \Rightarrow \sqrt{10}t = 0.8481$ or $\sqrt{10}t = 2.29353$<br>$t_1 = 0.2682,\ t_2 = 0.72527$<br>$\Rightarrow$ time = 0.457 (seconds) | | |
| | | **(4)** | |
| | | **(14 marks)** | |

**Notes:**

**(a)**

**M1:** Use of $T = \frac{\lambda x}{a}$

**M1:** Dependent on the preceding M1. Equate their tensions

**A1:** cao

**M1:** Condone sign error

**A1:** Correct unsimplified equation in $e$ and $x$ A1A1
Equation with one error A1A0

**A1:** Full working to justify conclusion that it is SHM about the equilibrium position

**(b)**

**B1ft:** Seen or implied. Follow their $e$

**M1:** Correct method for max. speed

**A1ft:** 1.81 or better. Follow their $a, \omega$

**(c)**

**B1:** Seen or implied

**M1:** Use of $x = a\cos wt$

**M1:** Correct strategy for the required interval

**A1:** 0.457 or better